"Lecture" Note
Engg. Mathematics-1
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Matrix

Chapter - 1

→ A rectangular arrangement of mn numbers in the form of 'm' horizontal rows 'n' vertical columns is called matrix.

[ ] bracket ( ) matrix
parenthesis

Any such matrix is usually written

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{vmatrix}$$

$a_{11}, a_{12}, a_{13}, \cdots\cdots a_{1n}$ – 1st row

$a_{21}, a_{22}, a_{23}, \cdots\cdots a_{2n}$ – 2nd row

$\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix}$ → 1st column $\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \\ \vdots \\ a_{n2} \end{pmatrix}$ → 2nd column

Types of matrix :-

1) Row matrix
2) column matrix
3) square matrix
4) Rectangular matrix
5) Identity matrix (unit matrix)
6) Null matrix (zero matrix)
7) Triangular matrix
8) U.T (Upper triangular)
9) L.T (Lower triangular)
10) skew symmetric matrix
11) symmetric matrix

1) Row matrix :-

→ A matrix having one row is called row matrix

Ex :- $(1\ 2\ 3)_{1\times3}$ – column

2) column matrix :-

→ A matrix having only one column is called column matrix

Ex :- $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}_{3\times1}$

3) <u>Square matrix</u> :-

→ The matrix having equal no. of rows to the equal no. of columns (2×2) is called square matrix

Ex $\begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2\times2}$ $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}_{3\times3}$

4) <u>Rectangular matrix</u> :-

→ The matrix having no. of rows not equal to the no. of columns.

Ex :- $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}_{\underset{\text{row}}{2} \times 3\text{ - column}}$ $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}_{3\times2}$

5) <u>Identity / unit matrix</u> :-

→ Diagonal elements have same number i.e equal to 1

Ex :- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times2}$ → Diagonal

6) <u>Zero / Null matrix</u> :-

→ Equal no. of elements having zero.

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3\times3}$ $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times2}$

7) <u>Symmetric matrix</u> :-

→ when $A = A^T$ then 'A' is said to be Symmetric matrix

Ex :- $\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}_{3\times3}$

$A^T = \begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}_{3\times3}$

8) Skew symmetric matrix :-

-> when $A = -A'$ (or) $A' = -A$ then A is called skew symmetric matrix.

Ex $A = \begin{pmatrix} 0 & h & g \\ h & 0 & f \\ g & f & 0 \end{pmatrix}$

$-A' = \begin{pmatrix} 0 & h & g \\ -h & 0 & -f \\ -g & -f & 0 \end{pmatrix}$

9) Triangular matrix :-

-> $\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$ -> lower triangular matrix

-> upper triangular matrix

*) Transpose :-

-> Transpose of a matrix 'A' is denoted by A^T (or) A' (or) A^T.

-> It is obtained by interchange of row and column.

(Row changes to column or column changes to row)

Ex :- $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3\times2}$ $A^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2\times3}$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2\times2}$ $A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}_{2\times2}$

Singular matrix :-

-> when $|A| = 0$ then it's called singular matrix

Non-singular matrix :-

when $|A| \neq 0$ then it's called non-singular matrix.

write down the matrix if $a_{ij} = 2i + 3j$

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$

$A = \begin{pmatrix} 2+3 & 2+6 & 2+9 \\ 4+3 & 4+6 & 4+9 \end{pmatrix}$ $A = \begin{pmatrix} 5 & 8 & 11 \\ 7 & 10 & 13 \end{pmatrix}$

ii) $a_{ij} = i \times j$

A) $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$

iii) $a_{ij} = i - j$

A) $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{pmatrix}$

iv) $a_{ij} = i/j$

A) $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \end{pmatrix}$

*) Adjoint of a matrix :-

→ Corresponding element of |A| of which the co-factors evaluated.

$$(A)_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$(A_{ij}) = (-1)^{i+j}$ m is minors

$$a_{11} = (-1)^{1+1} a_{11}$$
$$= (-1)^2 \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$$
$$= (a_{22} a_{33} - a_{32} a_{23})$$

$$a_{12} = (-1)^{1+2} a_{12}$$
$$= (-1)^3 \begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix}$$
$$= -(a_{21} a_{33} - a_{31} a_{23}$$

*) Inverse of matrix

$\boxed{A \quad B^{-1}}$

ii) $\boxed{A^{-1} = \frac{\text{Adj } A}{|A|}}$ → determinant of A

Inverse exist when it is non-singular

non-singular → $|A| \neq 0$

singular → $|A| = 0$

Step - 1 Find $|A|$

Step - 2 cofactors of matrix

Step - 3 Transpose

Step - 4 $A^{-1} = \frac{\text{Adj } A}{|A|}$

Q) $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{pmatrix}_{3\times 3}$

Ans) $\boxed{|A| = a_{1}(b_2c_3 - b_3c_2) - b_{1}(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)}$

$|A| = 3\begin{pmatrix} 0 & -1 \\ -5 & 0 \end{pmatrix} - (-1)\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} + (-2)\begin{pmatrix} 2 & 0 \\ 3 & 5 \end{pmatrix}$

$= 3(0-5) + 1(0-(-3)) + -2(10-0)$

$= -15 + 3 - 20 = -32$

Cofactor of $A_{11} = (-1)^2(0-5) = -5$

$A_{12} = (-1)^3(2-(-3)) = -5$

$A_{13} = (-1)^4(-10-3) = -13$

$A_{21} = (-1)^3(-20-10) = 10$

$A_{22} = (-1)^4(0-(-6)) = 6$

$A_{23} = (-1)^5(-10-0) = 10$

$A_{31} = (-1)^4(1-0) = 1$

$A_{32} = (-1)^5(-3-(-4)) = -1$

$A_{33} = (-1)^6(0-(-2)) = 2$

adj of A = [illegible]

inverse $A^{-1} = \frac{adj\ A}{|A|}$

Q) $A = \begin{pmatrix} x & -x \\ x & x^2 \end{pmatrix}$

ans) $\boxed{|A| = b_1c_2 - b_2c_1}$

$$|A| = x \times (x^2) - -x(x)$$
$$= x^3 - (-x^2)$$
$$= x^3 + x^2$$

cofactor of $A_1 = x^2$ $\quad A_{12} = -x$

$A_{21} = x$ $\quad A_{22} = x$

$$adj \text{ of } A = \begin{pmatrix} x^2 & x \\ -x & x \end{pmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|} = \frac{\begin{pmatrix} x^2 & x \\ -x & x \end{pmatrix}}{x^3 + x^2}$$

*) matrix method (system of linear eqn) :-

Let $Ax = B$

$x = B/A$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = B/A$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$$

$$a_{11}x + a_{12}y + a_{13}Z = 0$$
$$a_{21}x + a_{22}y + a_{23}Z = 0$$
$$a_{31}x + a_{32}y + a_{33}Z = 0$$

Soln = 0

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad B = \begin{pmatrix} d_1 \to 0 \\ d_2 \to 0 \end{pmatrix}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1) $x + y + z = 4$

$2x - y + 3z = 1$

$3x + 2y - z = 1$

ans) $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{pmatrix}$ $\quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad B = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$

$|A| = 1(1-6) - 1(-2-9) + 1(4+3)$

$= -5 + 11 + 7 = 13$

cofactors of

$A_{11} = (+1-6) = -5$ $\quad A_{21} = -(-1-2) = 3$ $\quad A_{31} = (3+1) = 4$

$A_{12} = -(-2-9) = 11$ $\quad A_{22} = -1-3 = -4$ $\quad A_{32} = -(3-2) = -1$

$A_{13} = (4+3) = 7$ $\quad A_{23} = -(2-3) = 1$ $\quad A_{33} = (-1-2) = -3$

adj of $A = \begin{pmatrix} -5 & 3 & 4 \\ 11 & -4 & -1 \\ 7 & 1 & -3 \end{pmatrix}$

$\therefore A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{\begin{pmatrix} -5 & 3 & 4 \\ 11 & -4 & -1 \\ 7 & 1 & -3 \end{pmatrix}}{13}$

$X = A^{-1}B$

$= \dfrac{1}{13} \begin{pmatrix} -5 & 3 & 4 \\ 11 & -4 & -1 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$

$= \dfrac{1}{13} \begin{pmatrix} -20 + 3 + 4 \\ +44 - 4 - 1 \\ 28 + 1 - 3 \end{pmatrix} = \dfrac{1}{13} \begin{pmatrix} -13 \\ 39 \\ 26 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

$\therefore x = -1 \quad y = 3 \quad z = 2$

Determinant

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$\boxed{x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}}$$

$$\boxed{y = \frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1}}$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$ (9 elements) → leading diagonal

This is a determinant of 3 order

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$ (4 - elements)

Leading diagonal

$$\boxed{\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}}$$ → laplace expansion

$= a_{11}(b_2c_3 - b_3c_2) - b_{11}(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

$= a_1b_2c_3 - a_1b_3c_2 - b_1a_2c_3 + b_1a_3c_2 + c_1a_2b_3 - c_1a_3b_2$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$\boxed{\Delta = a_1b_2 - a_2b_1}$$

minors

After deleting the rows and column of that elements the rest element is called minors

cofactors

$A_{ij} = (-1)^{i+j} M_{ij}$ $\quad$ $A_{11} = (-1)^{1+1} M_{11}$

$A_{12} = (-1)^{1+2} M_{12}$ $\quad$ $A_{21} = (-1)^{2+1} M_{21}$

$A_{22} = (-1)^{2+2} M_{22}$

Properties of Determinant :

1) The value of the determinant remains unchanged if row and column are interchanged

(i.e. $|A^T| = |A|$)

Example :

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

2) If any two rows (or) coloumn of a determinant interchanged then sign of determinant changes

Example

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

$R \longleftrightarrow R_3$ Then.

$$\begin{vmatrix} 1 & 5 & -7 \\ 6 & 0 & 4 \\ 2 & -3 & 5 \end{vmatrix} = -\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

3) If any two rows (or) coloumns of a determinant identical, the value of determinant is zero

Example

$$\begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix} = 0$$

because $R_1 = R_3$

[illegible]

[illegible]

[illegible]

Q) Solve $\begin{vmatrix} 16 & 19 & 13 \\ 15 & 18 & 12 \\ 14 & 17 & 11 \end{vmatrix}$

Ans) $\begin{vmatrix} 16 & 19 & 13 \\ 15 & 18 & 12 \\ 14 & 17 & 11 \end{vmatrix}$

$R_1 \to R_1 - R_2 \quad R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 14 & 17 & 11 \end{vmatrix}$$

$C_2 \to C_2 - C_3 \quad , \quad C_3 \to C_1 - C_3$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 14 & 6 & 3 \end{vmatrix} = 0$$

Q) Prove the following

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

Ans) $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = (-)\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$ (interchanging R_1 & R_2)

$= (-)(-)\begin{vmatrix} x & y & z \\ p & q & r \\ \end{vmatrix}$ (interchanging R_2 & R

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$$ (interchanging rows ...

$$= (-1)\begin{vmatrix} b & y & q \\ a & x & p \\ c & z & r \end{vmatrix}$$ interchanging $R_1 \leftrightarrow R_2$

$$= (-1)(-)\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$ interchanging C_1 ...

$$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} \quad \text{———} \text{(ii)}$$

Q) $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$ solve this

A)
$$abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$= \quad R_1 \to R_1 - R_2 \quad R_2 \to R_2 - R_3$

$$= abc\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= abc \times 1\begin{vmatrix} a-b & a^2-b^2 \\ b-c & b^2-c^2 \end{vmatrix}$$

$$= abc \times (a-b)(b-c)\begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix}$$

$$= abc\,(a-b)\,(b-c)\,(b+c-a-b)$$

$$= abc\,(a-b)\,(b-c)\,(c-a)$$

$C_2 \to C_2 - C_1$ [illegible]

$\begin{vmatrix} [illegible] \end{vmatrix}$ [illegible]

$= 1(3 - [illegible])$ [illegible]

$= 1 \times 1 = 1$

Q) $\begin{vmatrix} 3 & 1 & 3 \\ 0 & 4 & 0 \\ 1 & 3 & 1 \end{vmatrix} = 0$ True (or) False ?

→ It's true because its 2 Row columns C_1 & C_3 are identical.

Q) $\begin{vmatrix} 6 & 4 & 2 \\ 4 & 0 & 7 \\ 5 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 6 & 4 & 5 \\ 4 & 0 & 3 \\ 2 & 7 & 4 \end{vmatrix}$ True (or) false ?

→ It's true because it's Row & columns are interchanged.

⇒ CRAMER'S Rule :-

Step-1 :- x, y, z are the coefficient, of Determinant.

Step-2 Insted of x_1, x_2, x_3 Put the value of (B_1, B_2, B_3) i.e consta then find the determinant of Δ_x

Step-3 Insted of y_1, y_2, y_3 Put the value of (B_1, B_2, B_3) then find the Determinant of Δ_y

Step-4 Insted of z_1, z_2, z_3 Put the value of (B_1, B_2, B_3) then fin the Determinant of Δ_z.

Step-5 $x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$

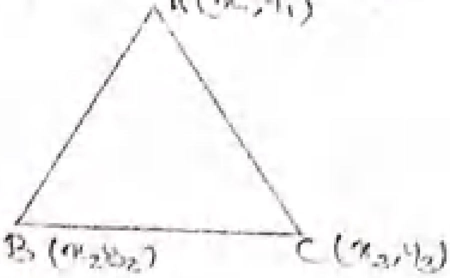
Some Imp Points :

i) 'n' no of linear eqn if $\Delta \neq 0$ then unique solution

ii) At least $\Delta_x = \Delta_y = \Delta_z = 0$ then it consistant solutio

1) Area of a Δ (triangle) given 3 sides,

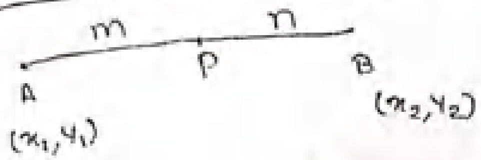
$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$



2) condition of colinearity :-

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

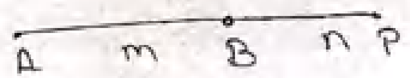
Internal & external division with (m:n ratio)



Internal division = $\bar{x} = \left(\frac{mx_2 + nx_1}{m+n}\right)$

$$\bar{y} = \left(\frac{my_2 + ny_1}{m+n}\right)$$

External division = $\bar{Q}$



$$\bar{x} = \frac{mx_2 - nx_1}{m-n}$$

$$\bar{y} = \frac{my_2 - ny_1}{m-n}$$

Distance :-

$P(x_1, y_1) \quad Q(x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

d Point

hen the Point is given, $P(x_1, y_1)$ $Q(x_2, y_2)$

$$\bar{x} = \frac{x_1 + x_2}{2}, \quad \bar{y} = \frac{y_2 + y_1}{2}$$

1) Slope intercept form :-

$$\boxed{y = mx + c}$$

2) When straight line passing through origin

$c = 0$

$$\boxed{y = mx}$$

3) Point - slope form :-

$$\boxed{y - y_1 = m(x - x_1)}$$

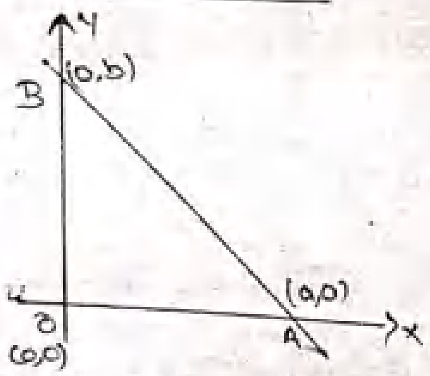
$P(x_1, y_1)$ slope $= m$

4) Two Point form :-

$P(x_1, y_1)$ & $Q(x_2, y_2)$

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

5) Intercept form :-



$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

a = x - intercept , b = y - intercept

6) origin (0,0) replace (x_0, y_0) when perpendicular

$$d = \left| \frac{c}{\pm\sqrt{a^2 + b^2}} \right|$$

Point of intersection

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$\boxed{x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}}$$

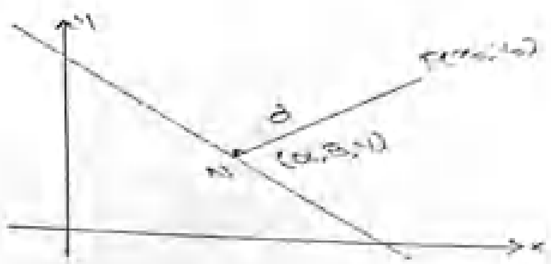
$$\boxed{y_1 = \frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1}}$$

Perpendicular distance of a point from a given line :-

$$\boxed{Ax + By + C = 0}$$

$P(x_0, y_0)$

$$\boxed{d = \left|\frac{ax_0 + by_0 + c}{\pm\sqrt{a^2 + b^2}}\right|}$$



i) when $\perp^r$ to each other ($L_1 \perp L_2$)

$$\boxed{a_1a_2 + b_1b_2 = 0}$$

ii) when $\parallel$ (parallel) ($L_1 \parallel L_2$)

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2}}$$

iii) co-incident if,

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

$\sin\alpha = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

$\cos\alpha = \frac{1}{\pm\sqrt{(1)^2 + (\sqrt{3})^2}}$

$= \frac{1}{\sqrt{1+3}}$

$= \frac{1}{\sqrt{4}} = \frac{1}{2}$

Reduction general to normal form :-

Normal $\quad x\cos\alpha + y\sin\alpha - P = 0$

$Ax + By + C = 0$

$$\boxed{\cos\alpha = \frac{a}{\pm\sqrt{a^2 + b^2}}}$$

$$\boxed{\sin\alpha = \frac{b}{\pm\sqrt{a^2 + b^2}}}$$

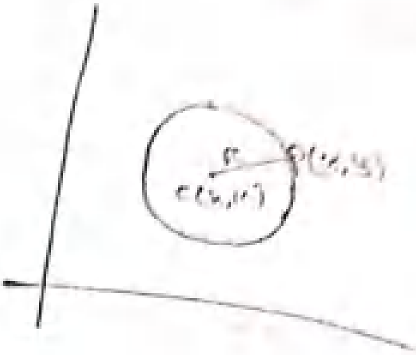
Circle

A circle is the locus of all points in a plane equidistant from a fixed point in the plane. The distance is called radius and the fixed point is called center.

Equation of the circle is given, center & radius

$CP^2 = r^2$

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$



When center is at origin (0,0)

$h = k = 0$

$\boxed{x^2 + y^2 = r^2}$

NB :- 1

1) $C(h,k)$ radius $= r$

& touches x-axis

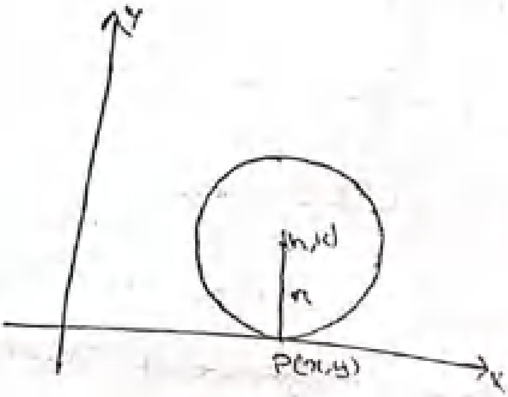
$CP^2 = r^2$

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-h)^2 + (y-k)^2 = k^2$

$\Rightarrow x^2 + h^2 - 2xh + y^2 + k^2 - 2ky = k^2$

$\Rightarrow \boxed{x^2 + h^2 - 2xh - 2ky + y^2 = 0}$



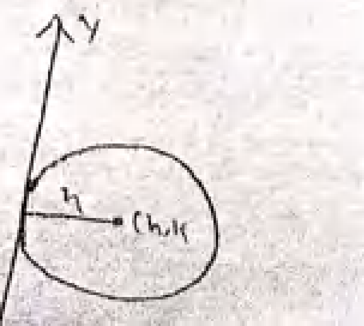
2) y-axis

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-h)^2 + (y-k)^2 = h^2$

$\Rightarrow x^2 + h^2 - 2xh + y^2 + k^2 - 2ky = h^2$

$\Rightarrow x^2 + y^2 + k^2 - 2xh - 2ky = \dots$



2) Find the eqn of the circle when it ... given $4x+y+3=0$, $x-2y+3=0$ & $x+y-1=0$, $4x+3y+7=0$

Ans) intersect point of two line

$x_1 = \frac{b_1c_2 - c_1b_2}{a_1b_2 - b_1a_2} = \frac{1\times7 - (-1)\times3}{1\times3 - 1\times4} = \frac{1+3}{3-4} = \frac{4}{-1} = -4$

$y_1 = \frac{c_1a_2 - a_1c_2}{a_1b_2 - b_1a_2} = \frac{1\times4 - 1\times7}{3-4} = \frac{-4-1}{-1} = \frac{-5}{-1} = 5$

another intersect point of two line

$4x + y + 3 = 0$

$x - 2y + 3 = 0$

$x_2 = \frac{b_1c_2 - c_1b_2}{a_1b_2 - b_1a_2} = \frac{1\times3 - 3\times(-2)}{4\times(-2) - 1\times1} = \frac{3+6}{-8-1} = \frac{9}{-9} = -1$

$y_2 = \frac{c_1a_2 - a_1c_2}{a_1b_2 - b_1a_2} = \frac{3\times1 - 4\times3}{4\times(-2) - 1\times1} = \frac{3-12}{-8-1} = \frac{-9}{-9} = 1$

eqn of circle is

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$\Rightarrow (x+4)(x+1) + (y-5)(y-1) = 0$

$\Rightarrow (x^2 + x + 4x + 4) + (y^2 - y - 5y + 5) = 0$

$\Rightarrow x^2 + 5x + y^2 - 6y + 9 = 0$

Find the eqn of circle ~~(3,2)~~ whose touches the line $x + 2y + 4 = 0$ & the center is $(3,2)$

$r = \left|\frac{lg + mf - n}{\sqrt{l^2 + m^2}}\right|$

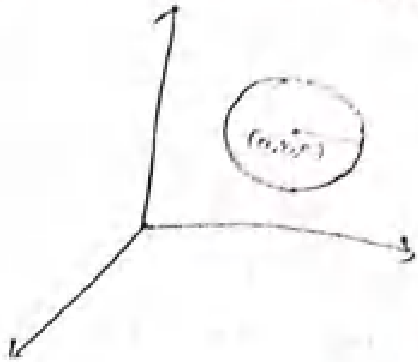
$= \left|\frac{1\times3 + 2\times2 - 4}{\sqrt{(1)^2 + (2)^2}}\right| = \left|\frac{3+4-4}{\sqrt{5}}\right| = \frac{3}{\sqrt{5}}$

$r^2 = \left(\frac{3}{\sqrt{5}}\right)^2 = \frac{9}{5}$

eqn of circle $(x-3)^2 + (y \ldots$

The Sphere

⇒ A sphere is a locus of a point in space which moves in such a way that it remain always constant [illegible] from a fixed point [illegible] radius of the sphere



The Eq^n of the sphere :-

center at point (a,b,c)

radius

$\cancel{r=}\ \boxed{r^2 = (x-a)^2+(y-b)^2+(y-c)^2}$

center at origin $(0,0,0)$

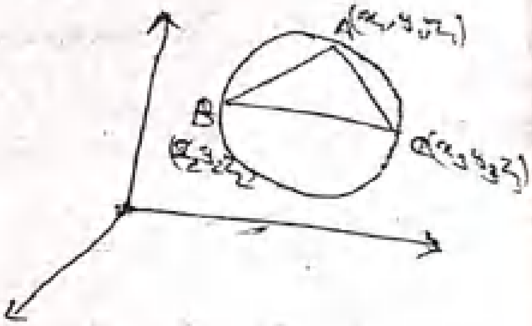
$$\boxed{x^2+y^2+z^2 = r^2}$$

General Eq^n of sphere :-

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$C(-u,-v,-w)$ $\quad d = \sqrt{(-u)^2+(-v)^2+(-w)^2}$

Diameter :-



$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0}$$

Q) Find the Eq^n of the sphere center $(3,1,-2)$ & sphere passing through $(1,1,2)$

A) $(x-a)^2+(y-b)^2+(z-c)^2 = r^2$

$\Rightarrow (x-3)^2+(y-1)^2+(z+2)^2 = r^2$

$\Rightarrow (1-3)^2+(1-1)^2+(2+2)^2 = r^2$

$(-2)^2+(0)^2+(4)^2 = r^2$

$4+16 = r^2$

$r^2 = 20$

$x^2+y^2+z^2-6x-2y+4z-20+14=0$

$x^2+y^2+z^2-6x-2y+4z-6=0$

d) passing through the points $(0,0,0)$ $(0,1,-1)$ $(-1,2,0)$ and $(1,2,3)$

A) Suppose eqⁿ of sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

Sphere passes through the $(0,0,0)$

$\therefore d=0$ —— (i)

Sphere passes through the pts $(0,1,-1)$ —— (ii)

$2v-2w=-2$

Sphere passes through the pt $(-1,2,0)$ —— (iii)

$-2u+4v=-5$

Sphere passes through $(1,2,3)$ —— (iv)

$2u+4v+6w=-14$

5) $P(4,5,-6)$ $Q(2,3,4)$ as a diameter find the eqⁿ of sphere

A) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$

$\Rightarrow (x-4)(x-2)+(y-5)(y-3)+(z+6)(z-4)=0$

$\Rightarrow x^2-2x-4x+8+y^2-3y-5y+15+z^2+6z-4z-24=0$

$\Rightarrow x^2+y^2+z^2-6x-8y+2z-1=0$

6) One end diameter $P(-1,2,4)$ find the co-ordinate of other end. eqⁿ of sphere $x^2+y^2+z^2-2x+4y-6z-7=0$

A) $2ux=-2x$ $\quad 2vy=4y$ $\quad 2wz=-6z$

$u=-1$ $\quad v=2$ $\quad w=-3$

$\therefore C(1,-2,3)$

$P(-1,2,4)$ $Q(x_2,y_2,z_2)$ & $C(1,-2,3)$

$x=\frac{x_1+x_2}{2}$ $\quad y=\frac{y_1+y_2}{2}$ $\quad z=\frac{z_1+z_2}{2}$

$\Rightarrow 1=\frac{-1+x_2}{2}$ $\quad \Rightarrow -2=\frac{2+y_2}{2}$ $\quad \Rightarrow 3=\frac{4+z_2}{2}$

$\Rightarrow 2=-1+x_2$ $\quad \Rightarrow -4=2+y_2$ $\quad \Rightarrow 6=4+z_2$

$\Rightarrow 2+1=x_2$ $\quad \Rightarrow -4-2=y_2$ $\quad \Rightarrow 6-4=z_2$

$\Rightarrow x_2=3$ $\quad \Rightarrow y_2=-6$